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Design of X-Bar Control Chart Using Multiple Dependent State Sampling Under Indeterminacy Environment

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ABSTRACT An X-bar control chart using the multiple dependent state (MDS) sampling under indeterminacy is presented in this paper. The MDS sampling utilizes the previous subgroup information if in-decision on the first sample. The use of MDS increases the power of the Shewhart control chart to detect a very small shift in the process. The advantages of the proposed chart over the past charts are given by simulation and real examples.

INDEX TERMS Uncertainty, indeterminacy, precise, interval value, imprecise, Neutrosophy.

I. INTRODUCTION

Every industrial process has natural variation and random variation. The control chart used to monitor these variations in the process [1]. Hart *et al.* [2] worked on the application of a control chart in healthcare. When the variable of interest is measurable, the Shewhart X-bar control chart is usually used to track the variation in the mean of the process. But, there is a need to enhance its power to track a small change in the process. The operational procedure such control chart is based on two control limits using the single sampling scheme. The process is declared as shifted if the plotting statistic cross the upper control limit (UCL) or lower control limit (LCL). The Shewhart control chart is designed for verity of reasons in the literature, see, for example, references [3]–[12].

As mentioned earlier, the Shewhart X-bar control chart using single sampling does not track a small shift in the process. Therefore, the use of multiple dependent state (MDS) sampling in control charts makes them powerful to detect a small shift in the process.

The MDS sampling utilizes the previous subgroup information if in-decision on the first sample. The control charts designed using MDS sampling are more efficient than charts using single sampling. The application of MDS sampling-based charts can be seen in [12]–[15].

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An approach that is used to compute the degrees of truth and falseness is known as fuzzy logic. The neutrosophic logic that is extension of fuzzy logic and computes the degrees of truth, falseness, and indeterminacy, see [16]–[20]. The Neutrosophic Statistics (NS) is the generalization of classical statistics (CS). The NS compute the measure of indeterminacy and applied when the data has imprecise or uncertain values. On the other hand, CS can be applied only when determined observations are available. Aslam and Khan [21] proposed X-bar chart using NS. More details about NS can be seen in [22] and [23]. Aslam [23] introduced NS in the area of statistical quality control. Some applications of control charts under NS can be seen in [21], [24]–[27].

The authors could not see work on X-bar control for MDS sampling under NS. In this paper, we will present the design of X-bar control for MDS sampling under NS. The efficiency of the proposed chart under uncertainty will be compared with [21] control chart in terms of neutrosophic average run length (NARL). We expect that the proposed chart will perform better than the existing chart under the uncertainty environment.

II. PROPOSED CONTROL CHART

Let $m_N \epsilon\, [m_L, m_U]$ and $\sigma_N \epsilon\, [\sigma_L, \sigma_U]$ represent the neutrosophic mean and standard deviation of neutrosophic random variable $X_{Ni} \epsilon\, [X_L, X_U]\,;\, i = 1, 2, 3, \ldots, n_N$ selected from the neutrosophic normal distribution. Let $\bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right]$

has the neutrosophic normal distribution with the neutrosophic sample mean $m_N \epsilon [m_L, m_U]$ and standard deviation $\sigma_N / \sqrt{n}_N$, where $n_N \epsilon [n_L, n_U]$ be neutrosophic sample size. Based on this information, the neutrosophic lower control limits (NLCL) and neutrosophic upper control limits (NUCL) are given by

$$LCL_{1N} = m_N - k_{1N} \frac{\sigma_N}{\sqrt{n}_N}; m_N \epsilon [m_L, m_U], \sigma_N \epsilon [\sigma_L, \sigma_U] \tag{1}$$

$$UCL_{1N} = m_N + k_{1N} \frac{\sigma_N}{\sqrt{n}_N}; m_N \epsilon [m_L, m_U], \sigma_N \epsilon [\sigma_L, \sigma_U] \tag{2}$$

$$LCL_{2N} = m_N - k_{2N} \frac{\sigma_N}{\sqrt{n}_N}; m_N \epsilon [m_L, m_U], \sigma_N \epsilon [\sigma_L, \sigma_U] \tag{3}$$

$$UCL_{2N} = m_N + k_{2N} \frac{\sigma_N}{\sqrt{n}_N}; m_N \epsilon [m_L, m_U], \sigma_N \epsilon [\sigma_L, \sigma_U] \tag{4}$$

where $k_{1N} \epsilon [k_{1L}, k_{1U}]$ and $k_{2N} \epsilon [k_{2L}, k_{2U}]$ are neutrosophic control limits coefficients associated with outer and inner control limits, respectively. The proposed chart using MDS sampling under neutrosophic statistical interval method (NSIM) is stated as follows

Step-1: Establish $LCL_{1N} \epsilon [LCL_{1L}, LCL_{1U}]$, $UCL_{1N} \epsilon [UCL_{1L}, UCL_{1U}]$, $LCL_{2N} \epsilon [LCL_{2L}, LCL_{2U}]$ and $UCL_{2N} \epsilon [UCL_{2L}, UCL_{2U}]$ for the in-control process.

Step-2: Select a neutrosophic random sample $X_{Ni} \epsilon [X_L, X_U]$ of size $n_N \epsilon [n_L, n_U]$ and compute $\bar{X}_N \epsilon [\bar{X}_L = \sum_{i=1}^{n_L} X_L / n_L, \bar{X}_U = \sum_{i=1}^{n_U} X_U / n_U]$.

Step-3: Declare the process in-control if $LCL_{2N} \epsilon [LCL_{2L}, LCL_{2U}] \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N} \epsilon [UCL_{2L}, UCL_{2U}]$. Otherwise, move to Step-4

Step-4: The process is in-control if $i_N \epsilon [i_L, i_U]$ processing subgroups are in-control, otherwise out-of-control.

Note here that based on Step-1 to Step-4, it can be noted that the proposed chart has two inner NLCL and two outer NLCL. According to these steps, the process is said to be an in-control state if the values of plotting statistic $\bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right]$ are within $LCL_{2N} \epsilon [LCL_{2L}, LCL_{2U}]$ and $UCL_{2N} \epsilon [UCL_{2L}, UCL_{2U}]$, otherwise, declare the process is out-of-control. If the experimenter is in-decision about the state of the process, then, the previous in-control subgroups are considered to take the decision about the state of the process. The process is declared in-control if $i_N \epsilon [i_L, i_U]$ previous subgroups are in-control.

The control chart proposed by [21] is the special case of the proposed control chart. The proposed control chart reduces to Shewhart X-bar chart if no uncertain observation is recorded from the production process. The neutrosophic probability of in-control when the process is at target, $m_N \epsilon [m_L, m_U]$, say $P_{in0N} \epsilon [P_{in0L}, P_{in0U}]$ is derived by following [13] as

$$P_{in0N} = P\left(LCL_{2N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N}\right) + P\left\{LCL_{1N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq LCL_{2N}\right\} + P\left\{UCL_{1N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N}\right\} \left\{P\left(LCL_{2N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N}\right)\right\}^{i_N}; P_{in0N} \epsilon [P_{in0L}, P_{in0U}]; i_N \epsilon [i_L, i_U] \tag{5}$$

Let $Z_N = \left(\bar{X}_N - m_N\right) / \left(\sigma_N / \sqrt{n}_N\right)$; $Z_N \epsilon [Z_L, Z_U]$ and $\Phi_{\mathrm{N}}(x_N)$; $\Phi_{\mathrm{N}}(x_N) \epsilon [\Phi_{\mathrm{L}}(x_L), \Phi_{\mathrm{U}}(x_U)]$ be a neutrosophic standard random variable and neutrosophic cumulative distribution function, respectively, see [22], [23]. The $P_{in0N} \epsilon [P_{in0L}, P_{in0U}]$ can be simplified as follows

$$P\left(LCL_{2N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N}\right) = P\left(\frac{LCL2_N - m_N}{\sigma_N / \sqrt{n}_N} \leq \frac{\bar{X}_N - m_N}{\sigma_N / \sqrt{n}_N} \leq \frac{UCL2_N - m_N}{\sigma_N / \sqrt{n}_N}\right); m_N \epsilon [m_L, m_U], \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \tag{6}$$

$$P\left(LCL_{2N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N}\right) = \Phi_{\mathrm{N}}(k_{2N}) - \Phi_{\mathrm{N}}(-k_{2N}) = 2\Phi_{\mathrm{N}}(k_{2N}) - 1 \tag{7}$$

Similarly

$$P\left\{LCL_{1N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq LCL_{2N}\right\} + P\left\{UCL_{1N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N}\right\} = 2\left[\Phi_{\mathrm{N}}(k_{1N}) - \Phi_{\mathrm{N}}(k_{2N})\right]; \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \tag{8}$$

The $P_{in0N} \epsilon [P_{in0L}, P_{in0U}]$ given in Eq. (5) can be written as

$$P_{in0N} = (2\Phi_{\mathrm{N}}(k_{2N}) - 1) + \{2[\Phi_{\mathrm{N}}(k_{1N}) - \Phi_{\mathrm{N}}(k_{2N})]\} \{(2\Phi_{\mathrm{N}}(k_{2N}) - 1)\}^{i_N}; P_{in0N} \epsilon [P_{in0L}, P_{in0U}]; i_N \epsilon [i_L, i_U] \tag{9}$$

The performance of the control chart is mostly measured by the neutrosophic average run length (NARL). The NARL using $P_{in0N} \epsilon [P_{in0L}, P_{in0U}]$ is given by

$$ARL_{0N} = \frac{1}{1 - \{(2\Phi_{\mathrm{N}}(k_{2N}) - 1) + \{2[\Phi_{\mathrm{N}}(k_{1N}) - \Phi_{\mathrm{N}}(k_{2N})]\}\{(2\Phi_{\mathrm{N}}(k_{2N}) - 1)\}^{i_N}\}}; ARL_{0N} \epsilon [ARL_{0L}, ARL_{0U}] \tag{10}$$

Suppose that $m_{1N} = m_N + c\sigma_N$; $m_{1N} \epsilon [m_{1L}, m_{1U}]$, $\sigma_N \epsilon [\sigma_L, \sigma_U]$ denotes the target mean for the shifted process, where c denotes the shift constant. The probability of in-control at $m_{1N} \epsilon [m_{1L}, m_{1U}]$, say $P_{in1N} \epsilon [P_{in1L}, P_{in1U}]$ is derived as follows

$$P\left(LCL_{2N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N} | m_{1N}\right) = P\left(\frac{LCL2_N - m_N}{\sigma_N / \sqrt{n}_N} \leq \frac{\bar{X}_N - m_N}{\sigma_N / \sqrt{n}_N} \leq \frac{UCL2_N - m_N}{\sigma_N / \sqrt{n}_N}\right); m_N \epsilon [m_L, m_U], \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \tag{11}$$

$$P\left(LCL_{2N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N}\right) = \Phi_{\mathrm{N}}\left(k_{2N} - c\sqrt{n}_N\right) + \Phi_{\mathrm{N}}\left(k_{2N} + c\sqrt{n}_N\right) - 1 \tag{12}$$

Similarly

$$P\left\{LCL_{1N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq LCL_{2N}\right\} + P\left\{UCL_{1N} \leq \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \leq UCL_{2N}\right\} = \Phi_{\mathrm{N}}\left(k1_N + c\sqrt{n}_N\right) - \Phi_{\mathrm{N}}\left(k_{2N} + c\sqrt{n}_N\right) + \Phi_{\mathrm{N}}\left(k_{1N} - c\sqrt{n}_N\right) - \Phi_{\mathrm{N}}\left(k_{2N} - c\sqrt{n}_N\right); \quad \bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right] \tag{13}$$

TABLE 1. The values of NARL when $n_N \epsilon$ [2, 5] and $i \epsilon$ [2, 4].

k_{1N}	[2.9821,3.2926]	[3.4265,3.6855]	[3.8032,3.8958]
k_{2N}	[2.1151,2.1354]	[2.0805,2.1896]	[2.0974,2.2258]
c	NARL		
0	[200.08,202.09]	[300.98,300.49]	[374.22,371.22]
0.01	[199.89,201.56]	[300.66,299.66]	[373.83,370.17]
0.02	[199.3,199.98]	[299.72,297.19]	[372.66,367.03]
0.05	[195.26,189.48]	[293.28,280.73]	[364.62,346.16]
0.08	[188.11,172.11]	[281.86,253.57]	[350.36,311.78]
0.1	[181.88,158.17]	[271.91,231.85]	[337.93,284.35]
0.2	[140.87,87.99]	[206.48,124.15]	[255.73,149.49]
0.3	[98.95,43.77]	[140.37,58.84]	[172.29,69.28]
0.4	[66.5,22.12]	[90.53,28.35]	[109.61,32.64]
0.5	[44.19,11.96]	[57.58,14.67]	[68.6,16.53]
0.6	[29.57,7.04]	[36.93,8.3]	[43.28,9.17]
0.7	[20.12,4.53]	[24.18,5.15]	[27.89,5.59]
0.8	[14,3.17]	[16.26,3.5]	[18.48,3.73]
0.9	[9.99,2.39]	[11.27,2.57]	[12.63,2.7]
0.95	[8.53,2.12]	[9.5,2.26]	[10.57,2.36]
1	[7.33,1.91]	[8.06,2.02]	[8.92,2.1]

The $P_{in1N} \epsilon [P_{in1L}, P_{in1U}]$ at $m_{1N} \epsilon [m_{1L}, m_{1U}]$ is given by

$$\begin{aligned} P_{in1N} = &\left(\Phi_N\left(k_{2N} - c\sqrt{n_N}\right) + \Phi_N\left(k_{2N} + c\sqrt{n_N}\right) - 1\right) \\ &+ \left\{\Phi_N\left(k_{1N} + c\sqrt{n_N}\right) - \Phi_N\left(k_{2N} + c\sqrt{n_N}\right)\right. \\ &+ \left.\Phi_N\left(k_{1N} - c\sqrt{n_N}\right) - \Phi_N\left(k_{2N} - c\sqrt{n_N}\right)\right\} \\ &\left\{\left(\Phi_N\left(k_{2N} - c\sqrt{n_N}\right) + \Phi_N\left(k_{2N} + c\sqrt{n_N}\right) - 1\right)\right\}^{i_N}; \\ &P_{in1N} \epsilon [P_{in1L}, P_{in1U}]; \quad i_N \epsilon [i_L, i_U] \end{aligned} \tag{14}$$

The NARL using $P_{in1N} \epsilon [P_{in0L}, P_{in1U}]$ is given by

$$ARL_{1N} = \frac{1}{1 - P_{in1N}}; \quad ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}], \quad P_{in1N} \epsilon [P_{in1L}, P_{in1U}] \tag{15}$$

The values of $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ for various $n_N \epsilon [n_L, n_U]$, $i_N \epsilon [i_L, i_U]$ and c and shown in Tables 1-3. From Tables 1-3, we note that for the same value of $i_N \epsilon [2, 4]$, the values of $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ decreases $n_N \epsilon [n_L, n_U]$ increases from $n_N \epsilon [2, 5]$ to $n_N \epsilon [8, 10]$.

The following algorithm under NSIM is implemented to find $k_{1N} \epsilon [k_{1L}, k_{1U}]$, $k_{2N} \epsilon [k_{2L}, k_{2U}]$ and $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$.

1. First of all fix the suitable values of $n_N \epsilon [n_L, n_U]$, $i_N \epsilon [i_L, i_U]$ and specified $ARL_{0N} \epsilon [ARL_{0L}, ARL_{0U}]$, say $r_{0N} \epsilon [r_{0L}, r_{0U}]$.
2. Determine $k_{1N} \epsilon [k_{1L}, k_{1U}]$ and $k_{2N} \epsilon [k_{2L}, k_{2U}]$ for which $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}] \geq r_{0N} \epsilon [r_{0L}, r_{0U}]$.
3. Several combinations of $k_{1N} \epsilon [k_{1L}, k_{1U}]$ and $k_{2N} \epsilon [k_{2L}, k_{2U}]$ exist and choose that one where $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ is very close to $r_{0N} \epsilon [r_{0L}, r_{0U}]$.
4. Determine $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ for various c.

III. ADVANTAGES OF THE PROPOSED CHART

The same values $n_N \epsilon [n_L, n_U]$ and c are considered to compare the performance of the proposed chart with [21] control chart. In control chart theory, it is well known that a chart having smaller values of $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ is called an efficient chart. The values of $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ when $r_{0N} \epsilon [200, 200]$, $r_{0N} \epsilon [300, 300]$ and $r_{0N} \epsilon [370, 370]$ for [21] control chart are shown in Table 4. The values of $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ when $i = [2, 4]$ are shown for the proposed control chart.

From Table 4, it can be charted that the proposed chart using MDS sampling under NSIM has smaller values of $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ as compared to [21] chart using single sampling under NSIM. It is worth to note that the proposed chart provides the smaller $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ for $r_{0N} \epsilon [200, 200]$, $r_{0N} \epsilon [300, 300]$ and $r_{0N} \epsilon [370, 370]$. For example, when $r_{0N} \epsilon [370, 370]$ and $c = 0.1$, the values of $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ from the proposed chart is $ARL_{1N} \epsilon [289.7, 259.39]$ and from [21] control chart it is $ARL_{1N} \epsilon [296.275, 274.526]$. We note a significant reduction in $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ as compared to [21]

TABLE 2. The values of NARL when $n_N \epsilon$ [5, 7] and $i \epsilon$ [2, 4].

k_{1N}	[3.2459,3.4177]	[2.9415,2.9431]	[3.095,3.2336]
k_{2N}	[2.0069,2.119]	[2.6841,2.7744]	[2.3239,2.3269]
c	ARL0		
0	[200.6,201.39]	[300.77,303.11]	[371.46,373.71]
0.01	[200.11,200.65]	[300.06,302.09]	[370.46,372.19]
0.02	[198.65,198.48]	[297.91,299.08]	[367.48,367.7]
0.05	[188.92,184.25]	[283.66,279.43]	[347.7,338.56]
0.08	[172.71,161.64]	[260.19,248.45]	[315.36,293.28]
0.1	[159.6,144.31]	[241.4,224.83]	[289.7,259.39]
0.2	[91.94,68.08]	[146.06,119.26]	[163.28,117.31]
0.3	[47.28,29.59]	[81.89,60.08]	[83.55,49.17]
0.4	[24.44,13.79]	[46.32,31.33]	[42.96,21.88]
0.5	[13.33,7.22]	[27.06,17.23]	[23.01,10.81]
0.6	[7.82,4.29]	[16.44,10.06]	[13.03,6]
0.7	[4.94,2.86]	[10.42,6.26]	[7.87,3.75]
0.8	[3.37,2.11]	[6.9,4.17]	[5.08,2.61]
0.9	[2.47,1.69]	[4.78,2.97]	[3.5,1.98]
0.95	[2.17,1.55]	[4.05,2.57]	[2.98,1.78]
1	[1.93,1.43]	[3.47,2.26]	[2.58,1.62]

control chart. From this study, we conclude that the proposed chart is more efficient than the existing control chart in $ARL_{1N} \epsilon\, [ARL_{1L}, ARL_{1U}]$.

To show the efficiency of the proposed chart over [21] control chart graphically, we used the simulated data generated from the in-control process when $X_{Ni} \epsilon\, [X_L, X_U]$ follows the neutrosophic normal distribution with $m_N \epsilon\, [0, 0]$ and $\sigma_N \epsilon\, [1, 1]$. Among the 40 observations, the first 20 observations are generated when $m_N \epsilon\, [0, 0]$ and next 20 observations are generated when $c = 0.5$. For the simulation study, we fixed $i = [2, 2]$, $n_N \epsilon\, [5, 5]$ and $r_{0N} \epsilon\, [370, 370]$. The table value for these parameters is $ARL_{1N} \epsilon\, [23.01, 10.81]$ which indicates that the control should detect the shift in the process from the 10$^{\text{th}}$ sample and 23$^{\text{rd}}$ sample. The values of statistic $\bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right]$ are computed and plotted in Figure 1 for the proposed control chart and on Figure 2 for [21] control chart. From Figures 1-2, we note that according to the expectation, the proposed chart indicates the shift in the process in a given indeterminacy interval. The proposed chart also shows several points in the in-decision indeterminacy interval which is required the practitioner's attention. For such points, the decision will be taken on the basis of the previous two subgroups. It means that at these points, the process is declared to be an in-control state if 2 previous subgroups are an in-control state. On the other hand, the existing control chart also indicates some points are in the in-decision area but it does not detect the shift in the process. From this study, we conclude that the proposed control chart is more efficient as compared to the existing control chart. Therefore, the proposed chart is effective and adequate to be applied in an uncertainty environment.

IV. CASE STUDY

Now the application of the proposed chart using MDS sampling under NSIM is given with the aid of automobile industry data. The inside diameter of engine piston rings is quality of interest and the practitioner wants to track it using the proposed control chart. Note here that the inside diameter is a continuous variable and measured with the help of some instrument. As mentioned by [28] "all observations and measurements of continuous variables are not precise numbers but more or less non-precise. This imprecision is different from variability and errors. Therefore also lifetime data are not precise numbers but more or less fuzzy. The best up-to-date mathematical model for this imprecision is so-called non-precise numbers". Therefore, it may possible that the recorded data have some neutrosophic numbers. The data having Neutrosophy cannot be analyzed using classical statistics. It is important to note that analyzing the inside diameter of engine piston rings data having a neutrosophic number using the classical statistics may mislead the experimenters or may increase the defective items of engine piston rings.

TABLE 3. The values of NARL when $n_N \epsilon$ [8, 10] and $i \epsilon$ [2, 4].

k_{1N}	[3.1302,3.5303]	[3.1254,3.4095]	[3.3928,3.585]
k_{2N}	[2.0376,2.1125]	[2.1834,2.2187]	[2.1444,2.2425]
c	NARL		
0	[200.33,203.39]	[300.06,304.45]	[375.43,370.46]
0.01	[199.54,202.34]	[298.79,302.73]	[373.79,368.33]
0.02	[197.2,199.24]	[295.02,297.69]	[368.93,362.02]
0.05	[182.01,179.44]	[270.69,265.71]	[337.48,322.13]
0.08	[158.31,149.79]	[233.31,218.75]	[289.12,263.71]
0.1	[140.49,128.52]	[205.65,185.75]	[253.34,222.8]
0.2	[65.02,49.44]	[92.31,67.95]	[108.46,78.86]
0.3	[28.2,18.73]	[39.01,24.58]	[43.39,27.58]
0.4	[13.13,8.22]	[17.62,10.29]	[18.74,11.21]
0.5	[6.82,4.32]	[8.82,5.15]	[9.1,5.48]
0.6	[4,2.69]	[4.96,3.06]	[5.01,3.19]
0.7	[2.63,1.92]	[3.12,2.11]	[3.12,2.17]
0.8	[1.92,1.53]	[2.19,1.63]	[2.17,1.66]
0.9	[1.53,1.3]	[1.68,1.37]	[1.66,1.38]
0.95	[1.4,1.23]	[1.52,1.28]	[1.5,1.29]
1	[1.3,1.17]	[1.4,1.21]	[1.38,1.22]

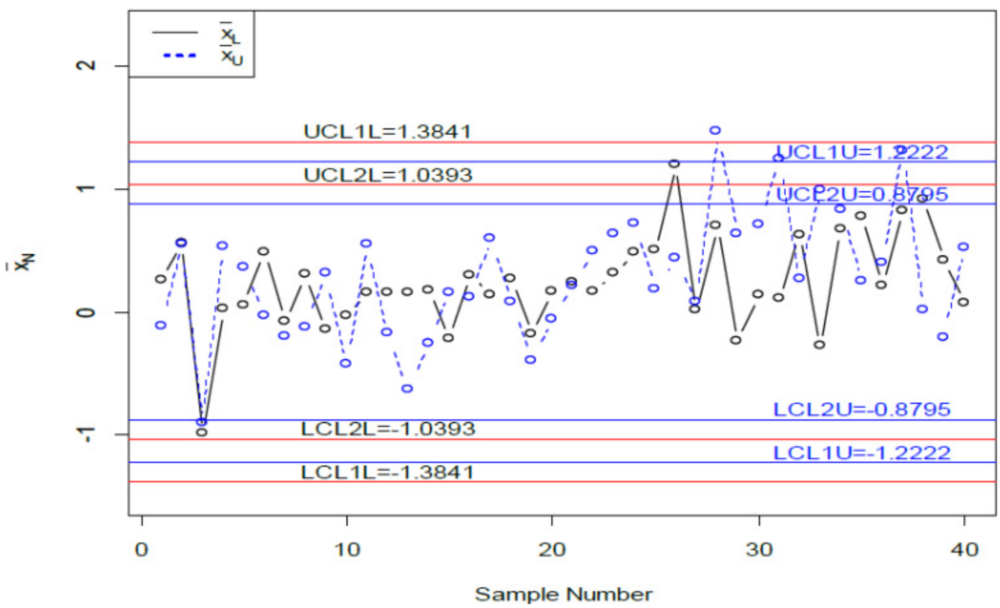

FIGURE 1. The control chart for the proposed chart using simulated data.

The same data given in Table 5 is used by [21] to design a chart using single a sampling scheme.

The values of statistic $\bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right]$ are given in the last column of Table 5. The neutrosophic control limits when $i = [2, 2]$, $n_N \epsilon$ [5, 5] and $r_{0N} \epsilon$ [370, 370] are given by

$$LCL1_N \epsilon\ [73.9871, 73.9881]\,; \sigma_N \epsilon\ [0.008896, 0.009399]\,,$$
$$m_N \epsilon\ [74.001, 74.001]\ UCL1_N \epsilon\ [74.01484, 74.01386]\,;$$

TABLE 4. Comaprision of two charts when $n_N \epsilon$ [5, 7] and $i \epsilon$ [2, 4].

c	Existing Control Chart			Proposed Control Chart		
	$r_{0N}\epsilon[200,200]$	$r_{0N}\epsilon[300,300]$	$r_{0N}\epsilon[370,370]$	$r_{0N}\epsilon[200,200]$	$r_{0N}\epsilon[300,300]$	$r_{0N}\epsilon[370,370]$
	ARL_{1N}			ARL_{1N}		
0	[200.353,201.421]	[300.105,302.326]	[371.082,372.596]	[200.6,201.39]	[300.77,303.11]	[371.46,373.71]
0.01	[199.917,200.808]	[299.397,301.326]	[370.17,371.314]	[200.11,200.65]	[300.06,302.09]	[370.46,372.19]
0.02	[198.62,198.987]	[297.29,298.363]	[367.458,367.516]	[198.65,198.48]	[297.91,299.08]	[367.48,367.7]
0.05	[189.951,187.037]	[283.258,279.007]	[349.429,342.764]	[188.92,184.25]	[283.66,279.43]	[347.7,338.56]
0.08	[175.549,167.983]	[260.138,248.483]	[319.851,303.953]	[172.71,161.64]	[260.19,248.45]	[315.36,293.28]
0.1	[163.906,153.27]	[241.62,225.196]	[296.275,274.526]	[159.6,144.31]	[241.4,224.83]	[289.7,259.39]
0.2	[103.143,85.323]	[147.48,120.865]	[178.023,144.66]	[91.94,68.08]	[146.06,119.26]	[163.28,117.31]
0.3	[60.386,45.292]	[83.84,62.047]	[99.695,73.029]	[47.28,29.59]	[81.89,60.08]	[83.55,49.17]
0.4	[35.753,24.983]	[48.32,33.185]	[56.672,38.458]	[24.44,13.79]	[46.32,31.33]	[42.96,21.88]
0.5	[21.931,14.577]	[28.891,18.798]	[33.443,21.463]	[13.33,7.22]	[27.06,17.23]	[23.01,10.81]
0.6	[14.016,9.018]	[18.013,11.301]	[20.588,12.718]	[7.82,4.29]	[16.44,10.06]	[13.03,6]
0.7	[9.341,5.91]	[11.718,7.203]	[13.227,7.992]	[4.94,2.86]	[10.42,6.26]	[7.87,3.75]
0.8	[6.487,4.094]	[7.949,4.858]	[8.864,5.318]	[3.37,2.11]	[6.9,4.17]	[5.08,2.61]
0.9	[4.689,2.992]	[5.617,3.461]	[6.19,3.739]	[2.47,1.69]	[4.78,2.97]	[3.5,1.98]
0.95	[4.045,2.606]	[4.792,2.979]	[5.251,3.198]	[2.17,1.55]	[4.05,2.57]	[2.98,1.78]
1	[3.523,2.299]	[4.129,2.596]	[4.499,2.771]	[1.93,1.43]	[3.47,2.26]	[2.58,1.62]

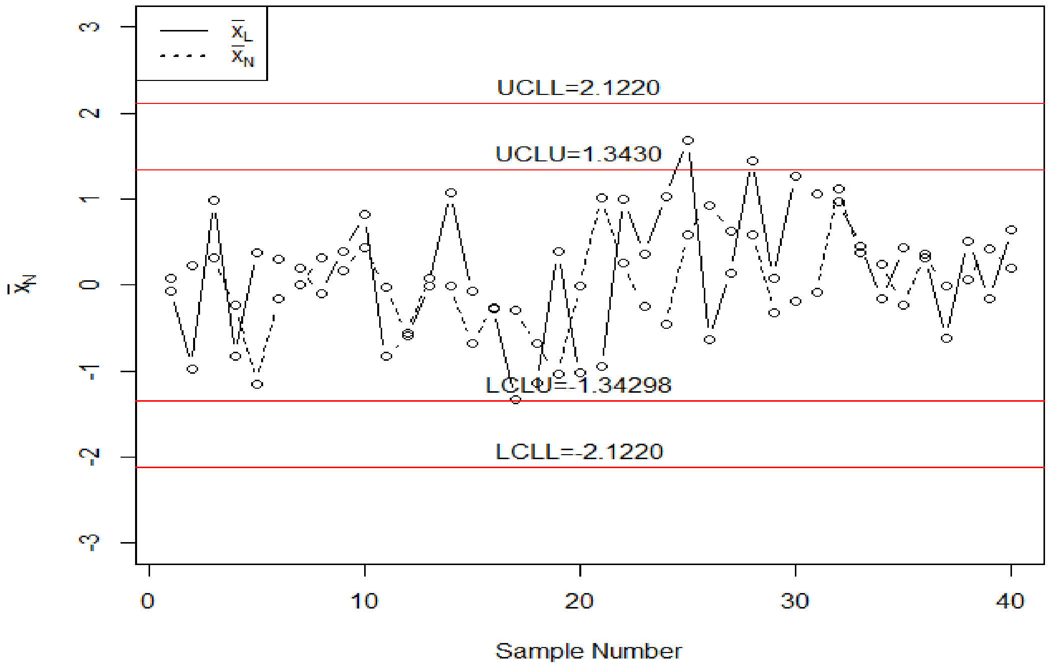

FIGURE 2. The control chart by [21] using simulated data.

$\sigma_N \epsilon$ [0.008896, 0.009399] , $m_N \epsilon$ [74.001, 74.001] $LCL2_N \epsilon$[73.99061, 73.99174] ; $\sigma_N \epsilon$ [0.008896, 0.009399], $m_N \epsilon$ [74.001, 74.001] $UCL2_N \epsilon$ [74.01139, 74.01026] ;

$\sigma_N \epsilon$ [0.008896, 0.009399] , $m_N \epsilon$ [74.001, 74.001]

The values of $\bar{X}_N \epsilon \left[\bar{X}_L, \bar{X}_U\right]$ plotted in Figure 3 for the proposed control chart and in Figure 4 for [21] chart.

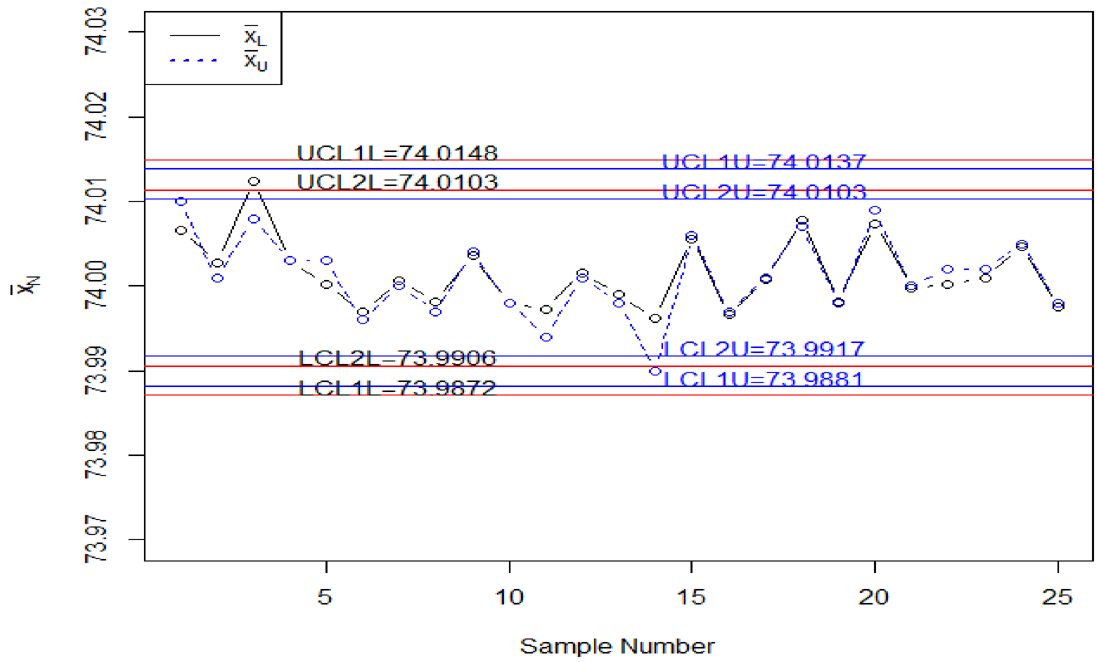

FIGURE 3. The proposed chart for real data.

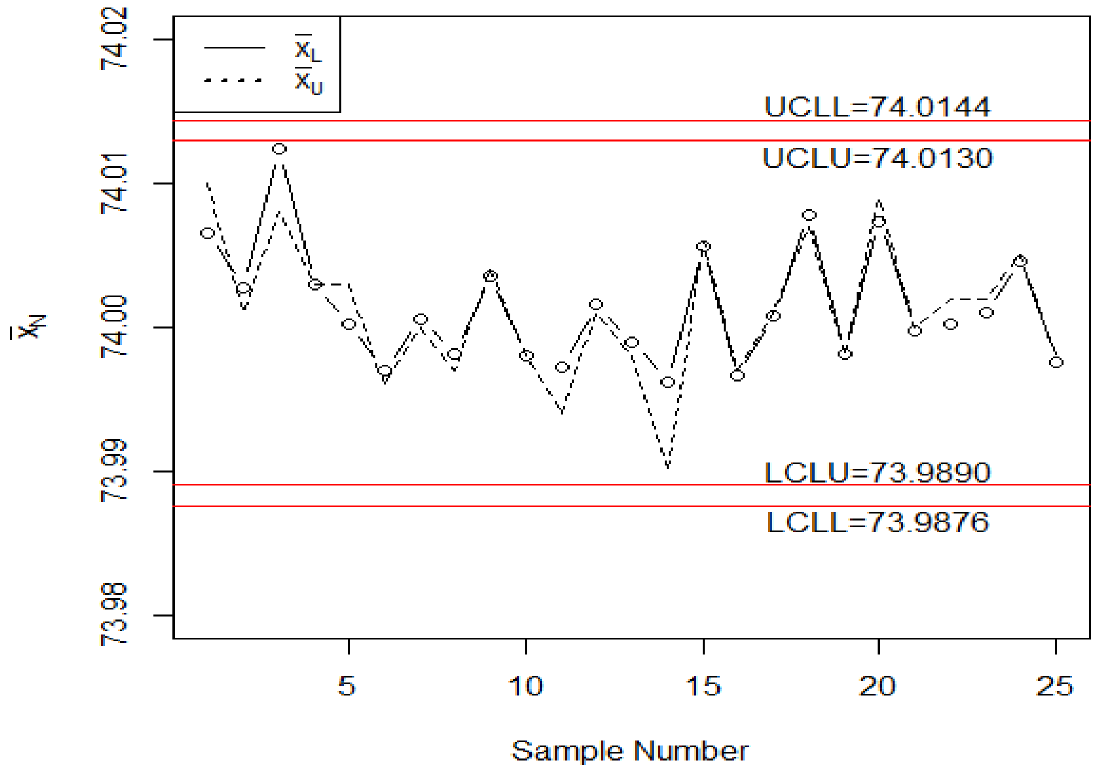

FIGURE 4. The chart by [21] for real data.

From Figure 3, it can be noted that 3 points are within the in-decision indeterminacy interval while the existing control chart shows the process is an in-control state. The points within the in-decision area can be interpreted as follows: the practitioner should observe the two previous subgroups to make the decision about the state of the process.

TABLE 5. The real example data [21].

	Sample Observation					$\bar{X}_N \epsilon [\bar{X}_L, \bar{X}_U]$
1	[74.03, 74.03]	[74.002, 73.991]	[74.019,74.0.19]	[73.992,73.992]	[74.008,74.001]	[74.0102,74.0066]
2	[73.995, 73.995]	[73.992, 74.003]	[74.001,74.001]	[74.011,74.011]	[74.004,74.004]	[74.0006,74.0028]
3	[73.988, 74.017]	[74.024, 74.024]	74.021,74.021]	[74.005,74.005]	[74.002,73.995]	[74.008,74.0124]
4	[74.002, 74.002]	[73.996, 73.996]	[73.993,73.993]	[74.015,74.015]	[74.009,74.009]	[74.003,74.003]
5	[73.992, 73.992]	[74.007, 74.007]	[74.015,74.015]	[73.989,73.989]	[74.014,73.998]	[74.0034,74.0002]
6	[74.009, 74.009]	[73.994, 74.001]	[73.997,73.997]	[73.985,73.985]	[73.993,73.993]	[73.9956,73.997]
7	[73.995, 73.998]	[74.006, 74.006]	[73.994,73.994]	[74,74]	[74.005,74.005]	[74,74.0006]
8	[73.985, 73.985]	[74.003,74.01]	[73.993,73.993]	[74.015,74.015]	[73.988,73.988]	[73.9968,73.9982]
9	[74.008, 74.005]	[73.995, 73.995]	[74.009,74.009]	[74.005,74.005]	[74.004,74.004]	[74.0042,74.0036]
10	[73.998, 73.998]	[73.998,73.998]	[73.99,73.99]	[74.007,74.007]	[73.995,73.995]	[73.998,73.998]
11	[73.994, 73.998]	[74,74.002]	[73.994,73.994]	[73.995,73.995]	[73.99,74.001]	[73.9942,73.9972]
12	[74.004, 74.004]	[74,74.002]	[74.007,74.005]	[74,74.001]	[73.996,73.996]	[74.0014,74.0016]
13	[73.983, 73.993]	[74.002,74.002]	[73.998,73.998]	[73.997,73.997]	[74.012,74.005]	[73.9984,73.999]
14	[74.006, 74.006]	[73.967,73.985]	[73.994,73.994]	[74,74]	[73.984,73.996]	[73.9902,73.9962]
15	[74.012, 74.012]	[74.014,74.012]	[73.998,73.998]	[73.999,73.999]	[74.007,74.007]	[74.006,74.0056]
16	[74, 74]	[73.984,73.984]	[74.005,74.005]	[73.998,73.998]	[73.996,73.996]	[73.9966,73.9966]
17	[73.994, 73.994]	[74.012,74.012]	[73.986,73.986]	[74.005,74.005]	[74.007,74.007]	[74.0008,74.0008]
18	[74.006, 74.006]	[74.01,74.011]	[74.018,74.018]	[74.003,74.003]	[74,74.001]	[74.0074,74.0078]
19	[73.984, 73.984]	[74.002,74.002]	[74.003,74.003]	[74.005,74.005]	[73.997,73.997]	[73.9982,73.9982]
20	[74,74]	[74.01,74.01]	[74.013,74.009]	[74.02,74.015]	[74.003,74.003]	[74.0092,74.0074]
21	[73.982, 73.982]	[74.001,74.001]	[74.015,74.015]	[74.005,74.005]	[73.996,73.996]	[73.9998,73.9998]
22	[74.004, 74.004]	[73.999,73.999]	[73.99,73.99]	[74.006,74.006]	[74.009,74.002]	[74.0016,74.0002]
23	[74.01, 74.01]	[73.989,73.989]	[73.99,73.99]	[74.009,74.005]	[74.014,74.011]	[74.0024,74.001]
24	[74.015, 74.011]	[74.008,74.008]	[73.993,73.993]	[74,74]	[74.01,74.011]	[74.0052,74.0046]
25	[73.982, 73.982]	[73.984,73.989]	[73.995,73.995]	[74.017,74.012]	[74.013,74.01]	[73.9982,73.9976]

V. CONCLUDING REMARKS

In this paper, the X-bar control chart using MDS sampling under NSIM is presented. The probabilities of in-control and out-of-control processes are derived. Some necessary tables are given for industrial use. The neutrosophic algorithm is also given to determine the values of NARL. The comparison of the proposed chart is given over the existing chart with the aid of NARL, simulation, and real example. From this comparison, we concluded that the proposed chart performs better in detecting a shift in the process as compared to the existing chart. We recommend the use of the proposed chart in the industries for the monitoring of complex processes or where uncertainty is found in recording the data. The proposed control chart for the big data is a fruitful area of future research. The proposed chart using the ranked set sampling can be considered as future research. The proposed chart using some non-normal distributions is also a fruitful area for future research.

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